Exp vs Theory?

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chemical potential

$\alpha = \frac{-\mu}{k_B T}$

per quantum state

Bose distribution $n(\mathcal{E}) = \frac{1}{e^{\alpha} e^{\mathcal{E}/k_B T} - 1}$

# of states in $d\mathcal{E}$

Number of bosons (He atoms) $N = \int_0^\infty n(\mathcal{E}) \rho(\mathcal{E}) d\mathcal{E}'$ (fixed)

$$\Rightarrow N = \int_0^\infty \frac{\rho(\mathcal{E}) d\mathcal{E}}{e^{\alpha} e^{\mathcal{E}/kT} - 1}$$

V: volume of box

$$\rho(\mathcal{E}) d\mathcal{E} = \frac{4\pi V}{h^3} \sqrt{2m^3} \sqrt{\mathcal{E}}\, d\mathcal{E}$$ (solid-state 1 or 2)

$$\Rightarrow N = \frac{(2\pi m k_B T)^{3/2} V}{h^3} e^{-\alpha} \left(1 + \frac{e^{-\alpha}}{2^{3/2}} + \frac{e^{-2\alpha}}{3^{3/2}} + h.c.\right)$$

consider high T and N fixed $\rightarrow e^{-\alpha}$ is small $\rightarrow$ only 1st term

$$N = \frac{(2\pi m k_B T)^{3/2} V e^{-\alpha}}{h^3} \rightarrow e^{-\alpha} = \frac{N h^3}{(2\pi m k_B T)^{3/2} V}$$

(classical Bolzmann)

Total energy of system $E = \int_0^\infty \mathcal{E}\, n(\mathcal{E}) \rho(\mathcal{E})\, d\mathcal{E}$

$$E = \frac{(2\pi m k_B T)^{3/2} V}{h^3} \left(\frac{3}{2} k_B T\right) e^{-\alpha} \left(1 + \frac{e^{-\alpha}}{2^{5/2}} + \frac{e^{-2\alpha}}{3^{5/2}} + h.c.\right)$$

high T $\rightarrow$ only 1st term $\rightarrow E = 3/2\, N k_B T$

average $\bar{E} = \frac{E}{N} = 3/2\, k_B T$ (equipartition theorem)

Low T: include up to $e^{-2\alpha}$

$$\bar{E} = \frac{3}{2} k_B T \left(1 - \frac{1}{2^{5/2}} \frac{N h^3}{(2\pi m k_B T)^{3/2} V}\right)$$

Bose condensation term

lower $\bar{E}$ as classical

When do quantum effects play a role?

Distance d comparable to de Broglie w.l. λ

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{3 m k_B T}} \geq d \qquad \left(mv^2 = \frac{3}{2} k_B T\right)$$

$$d^3 = \frac{V}{N} \longrightarrow \frac{h^3}{(3 m k_B T)^{3/2}} \geq \frac{V}{N} \Longrightarrow \frac{N h^3}{(3 m k_B T)^{3/2} V} \geq 1$$

$[2^{5/2} \approx 5.6]$

$$\Longrightarrow \frac{N h^3}{(2\pi m k_B T)^{3/2} V} \gtrsim 1/3$$

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$$\frac{N h^3}{(2\pi m k T)^{3/2} V} \gtrsim \frac{1}{3} \qquad \frac{N}{V} = \bar{n} \quad \text{(density)}$$

$$(2\pi m k T_c)^{3/2} = 3 n h^3 \longrightarrow 2\pi m k_B T_c = 3^{2/3} n^{2/3} h^2$$

$$T_c = \frac{3^{2/3}}{2\pi} \frac{n^{2/3} h^2}{m k_B} \simeq \frac{3.31\, h^2}{m k_B} n^{2/3}$$

^4He boson, $m = 6.69 \times 10^{-24}\,g$, $\bar{n} = 2.2 \times 10^{22}\,cm^{-3}$ (liquid He)

$\Rightarrow T_c \simeq 3.14\,K$ Experiment: $T_c = 2.2\,K$

N_2 : $\lambda \simeq 0.03\,Å$ (mass) $d \simeq 3 \times 10^{-9}\,m = 30\,Å \Rightarrow$ classical
at liquid N_2 T

e^- in solid : $\lambda \simeq 1\,nm$ $d \simeq 1\,Å \Rightarrow$ quantum gas

Now superconductivity

Observations:
* $R = 0$ (supercurrents > age of universe)
* Meisner effect $\Rightarrow$ new quantum state
* Flux quantisation

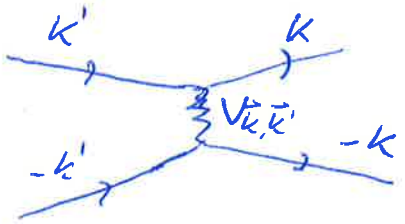
BCS theory:

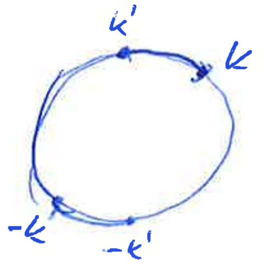
Impose attractive potential due to instability at E_F

$$H_{BCS} = \sum_k \underset{\substack{\uparrow \\ \text{usual band}}}{\xi_k} c^{\dagger}_{k\sigma} c_{k\sigma} - \sum_k \left(\Delta_k c^{\dagger}_{-k\downarrow} c^{\dagger}_{k\uparrow} + h.c. \right)$$

↳ energy to pay to keep $2e^-$ together

Attraction due to electron-phonon-electron interaction





Consequences of BCS

- gap in single e^- DOS $\Delta = 3.53\, k_B T_c$
- ~~$T_c \simeq \Theta_D$~~ $T_c = 1.13\, \Theta_D\, e^{(-1/(D(E_F)\lambda_{e\text{-}ph}))} = \Theta_D\, e^{(-1/(\lambda - \mu^*))}$

Θ_D : Debye energy (highest E phonon)

$\lambda_{e\text{-}ph}$: electron-phonon coupling strength

$D(E_F)$: DOS at E_F μ^* : effective Coulomb repulsion

Eliashberg theory : $\lambda = 2 \int_0^\infty \underbrace{\alpha^2 F(\omega)}_{\text{Eliashberg function}} \frac{d\omega}{\omega}$

- phonons depend on mass $\longrightarrow$ isotope effect in T_c

* How to measure gap ?
* " e^--phonon interaction λ $\alpha^2 F(\omega)$?
* why $R = 0$? $\hbar \vec{k}_p = 0 \rightarrow \lambda \rightarrow \infty$ (whole condensate) $\psi(r) = \sqrt{n_s(r)}\, e^{i\varphi(\vec{r})}$

High Tc superconductors (cuprates etc.)

+ BCS formula for T_c does not work
* No (?) isotope effect
* Symmetry of gap function d-wave vs. s-wave

coupling due to spin fluctuations ? s repulsive d attractive

Current : $j = q\, \psi^* v\, \psi$ with $v = \frac{-i\hbar}{m} \nabla$

$\rightarrow j_s \approx \frac{-iq\hbar}{2m} (\psi^* \nabla \psi - \psi \nabla \psi^*)$ + h.c.

$= \frac{-iq\hbar}{2m} n_s (e^{-i\varphi} e^{i\varphi} i \nabla \varphi + i e^{i\varphi} e^{-i\varphi} \nabla \varphi) = + \frac{q\hbar}{2m} n_s\, 2 \nabla \varphi$

whole condensate + phase quantisation

High Tc S.C.

- Low conductivity above T_c
- Non Drude like behaviour in optical conductivity.

$\Longrightarrow$ non-Fermi liquid

$\Longrightarrow$ Bands at E_F are <u>not</u> single particle bands

- ZRS shared between unit cells

Higher energy scale (eV) $\Longrightarrow$ faster ($<$ fs) + smaller

- Condense ZRS below Tc. what is competing E-scale ?? !

<u>coupling mechanism ???</u>

? Anti-ferromagnetic fluctuations:

(Bare four Fermion) <u>interaction</u>):

$$\Gamma^{0}_{\alpha\gamma,\beta\delta}(q) = -g^2 \chi_0(q)\,(T_{\alpha\gamma,\beta\delta} - 3 S_{\alpha\gamma,\beta\delta})$$

Triplet   singlet

$T_{\alpha\gamma,\beta\delta} = (\delta_{\alpha\beta}\delta_{\gamma\delta} + \delta_{\alpha\delta}\delta_{\beta\gamma})/2$ $\delta \hat{=}$ expanded Pauli matrices σ

$S_{\alpha\gamma,\beta\delta} = (\delta_{\alpha\beta}\delta_{\gamma\delta} - \delta_{\alpha\delta}\delta_{\beta\gamma})/2$

Repulsive for s-wave

attractive for d-wave